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EVALUATION OF DETERIORATING INVENTORY MODEL WITH TIME DEPENDENT DEMAND AND PARTIAL BACKLOGGING

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ABSTRACT

We developed an inventory model for deteriorating items under time dependent demand function. Inventory holding cost is a linear function of time. For deterioration of units we considered two parameters Weibull distribution. In this paper, a more realistic scenario is assumed where the part of shortages was backordered and the rest was lost with time. The backordering rate is variable and it is considered exponential function depending on waiting time for the next replenishment. Solution procedure of the developed model is presented with numerical example and its sensitivity analysis. In most of the cases the demand for items increases with time and the items that are stored for future use always loose part of their value with passage of time. In inventory this phenomenon is known as deterioration of items. The rate of deterioration is very small in some items like hardware, glassware, toys and steel. The items such as medicine, vegetables, gasoline alcohol, radioactive chemicals and food grains deteriorate rapidly over time so the effect of deterioration of physical goods cannot be ignored in many inventory systems. The deterioration of goods is a realistic phenomenon in many inventory systems and controlling of deteriorating items becomes a measure problem in any inventory system. Due to deterioration the problem of shortages occurs in any inventory system and shortage is a fraction that is not available to satisfy the demand of the customers in a given period of time.

Keywords : Inventory model, deteriorating items, Time dependent demand; shortages

INTRODUCTION

Deterioration of produced items is very important factor in inventory management and in any production. In the recent time more research work has been carried out in the inventory management to develop inventory models with deterioration of items and shortages are permitted. Commodities like vegetables, fruits and food items from depletion by direct spoilage while kept in store. It has been observed that the failure of many items may be expressed by Weibull distribution.

Ghare and Scharder [2018] first formulated a mathematical model with a constant deterioration rate. Khemlnitsky and Gerchak [2015] developed optimal control approach to production systems with inventory level dependent demand. Ruxian , Hongjie Lan and John Mawhinney [2012] reviewed on deteriorating inventory study. Vipin Kumar, Singh and Sanjay Sharma [2017] developed inventory model for profit maximization production with time dependent demand and partial backlogging. Papachristos and Skouri [2014] gave an inventory model with deteriorating items, quantity discount, pricing and time-dependent partial backlogging. Samanta and Ajanta Roy [2011] presented a production inventory model with deteriorating items and shortages. Inventory model with price dependent demand and time varying holding cost was given by Ajantha Roy [2011]. Vinod Kumar Mishra and Lal Sahab Singh [2010] have given deteriorating inventory model for time dependent demand and holding cost with partial backlogging. Kirtan Parmar, Indu Aggarwal and Gothi [2016] have formulated an order level inventory model for deteriorating items under varying demand condition. Patel [2012] developed the first deteriorating inventory model with linear trend in demand. He considers demand as a linear function of time. Nahmias [2005] gave a review on perishable inventory theory. He reviewed the relevant literature on the problem of determining suitable ordering policies for both fixed life perishable inventory, and inventory subjected to continuous exponential decay. Rafaat [2017] gave a survey of literature on continuously deteriorating inventory models and he considered the effect of deterioration as a function of the on hand level of inventory. He focused to present an up-to-date and complete review of the literature for the continuously deteriorating mathematical inventory models. But all researchers assume that during shortage period all demand either backlogged or lost. In reality it is observed that some customers are willing to wait for the next replenishment.

PARTIAL BACKLOGGING

Supply chain system in which a supplier prepares for the selling season by building stock levels prior to the beginning of the season and shortages realized at the beginning of the season are represented as mixtures of backorders and lost sales. The many real-life situations, the practical experiences reveal that some but not all customers will wait for backlogged items during a shortage period, such as for fashionable commodities or high-tech products with short product life cycle. The longer the waiting time is, the smaller the backlogging rate would be. According to such phenomenon, taking the backlogging rate into account is necessary.

Order Level : The level of stock of any item at which an order is initiated for more supplies of that item.

Safety stock : Safety stock is a term used by inventory specialists to describe a level of extra stock that is maintained below the cycle stock to buffer against stock outs. Safety Stock (also called Buffer Stock) exists to counter uncertainties in supply and demand. Safety stock is defined as extra units of inventory carried as protection against possible stock outs (shortfall in raw material or packaging). By having an adequate amount of safety stock on hand, a company can meet a sales demand which exceeds the demand they forecasted without altering their production plan. It is held when an organization cannot accurately predict demand and/or lead time for the product. It serves as an insurance against stock outs.

Quantity Discount and Order Quantity : Form of an economic order quantity (EOQ) model that takes into account quantity discounts. Quantity discounts are price reductions designed to induce large orders. If quantity discounts are offered, the buyer must weigh the potential benefits of reduced purchase price and fewer orders against the increase in carrying costs caused by higher average inventories. Hence, the buyer's goal in this case is to select the order quantity that will minimize total costs, where total cost is the sum of carrying cost, ordering cost, and purchase cost.

It is a common business practice for pricing schedule to display economies of scale with prices decreasing as lot size increases. Such pricing schedules offer discounts based on the quantity ordered in a single lot. This encourages the retailers to order in larger lots to take advantage of price discounts. This adds to the average inventory and flow time in a supply chain. Unlike the EOQ model, the purchase cost now becomes an important criterion in determining the optimal order size and the corresponding total annual inventory cost.

Sometimes, manufacturers use trade promotions to increase sales by offering a discounted price over a pre specified period of time over which the discount is effective. In some cases, the manufacturer may require some specific actions from the retailer to qualify for the discount, such as, displays, advertising, promotion, and so on. Trade promotions are quite common in the consumer packaged goods industry, with manufacturers promoting different things at different times of the year. The goal of the trade promotions is to influence retailers to act in a way that helps the manufacturer achieve its goal.

Deterioration : In the inventory management, the decay of the items plays an important role. In reality, some of the items are either damaged or decayed or vaporized or affected by some other factors, i.e. these are not in a perfect condition to satisfy the demand. The rate of the deterioration of an item is either constant, time dependent or stock dependent. Some items which are made of glass, China clay or ceramic break during their storage period for which the deterioration rate depends upon the size of the total inventory. The decaying item such as photographic film, electronic goods, fruits and vegetables gradually loose their utility with time. Deterioration is defined as change, damage, decay, spoilage, evaporation, obsolescence, pilferage, and loss of utility or loss of marginal value of a commodity that results in decreasing usefulness from the original one. Most products such as medicine, blood, fish, alcohol, gasoline, vegetables and radioactive chemicals have finite shelf life, and start to deteriorate once they are replenished.

Trade credit : The traditional economic order quantity (EOQ) model focuses on the buyer's view and makes several assumptions, for example, no stock-outs, fixed demand rate, unlimited store space, zero lead time and must be paid for the items as soon as the items were received. But we know these assumptions are rarely met in real-life situation. For instance, in most business transactions, the supplier would allow a specified credit period (say, 30 days) to the retailer for payment without penalty to stimulate the demand of his/her products. This credit term in financial management is denoted as "net 30." Before the end of the trade credit period, the retailer can sell the goods and accumulate revenue and earn interest.

Warehouse : A warehouse is a location with adequate facilities where volume shipments are received from a production centre, broken order or orders, and shipped to the customer's location or locations. The rationale for establishing a warehouse in a distribution network is the creation of a differential advantage for the firm.

The concept of a distribution warehouse or a distribution centre is vastly different from the earlier concept of a godown for storage. The godown is merely a dumping place. Godowns are maintained merely for storage of surplus place. The earlier concept, which led to the establishment of warehouses, was based on the need for ensuring a continuous, uninterrupted supply of goods in the market area for the following:

- Ensuring protection against delays and uncertainties in transportation arising from a variety of factors;
- Eliminating lack of sophistication in production control and consequent uncertainties in the availabilities of product at the desired time and place;
- Providing for adjustment between the time of production and the time of use because production and use can be synchronized;
- Serving as a reservoir of goods, receiving surplus goods when production exceeds demand and releasing in anticipated.

From the foregoing, it is obvious that earlier a warehouse considered a necessary evil which was to be toletated, but which did little to provide a differential advantage. The modern distribution centre or distribution warehouse is a pivot in the physical distribution system. According to this system, movement is the primary objective of a warehouse. As per this new concept, a warehouse is a location where inputs (incoming factory shipments) are converted into outputs (outputs shipments representing orders of customers). This conversion takes place without consuming too much time. The goods may be received over a period of time from different places, combined or broken down into each individual customer's orders, and dispatched to the next point in the distribution channel without their coming rest within the confines of the distribution centre. Because of the usual and often inevitable lack of co-ordination between inbound and outbound goods, storage facilities of a temporary nature must be provided for in the scheme. However, the distribution centre continues to be a dynamic location in which flow is accentuating and where storage, with its static connotation, is a facilitating function of secondary importance. Conceptually, the distribution centre is not unlike a retail store in its interactions with its customers.

ASSUMPTIONS AND NOTATIONS

To develop an inventory model with variable demand and partial backlogging the following notations and assumptions are used:

- ❖ Assumptions
- ❖ Demand rate is taken as linear.
- ❖ Deterioration rate is time dependent.
- ❖ Shortages are allowed with partial backlogging.
- ❖ Backlogging rate is an exponential decreasing function of time.
- ❖ Replenishment rate is infinite.
- ❖ A single item is considered over the prescribed interval.
- ❖ There is no repair or replenishment of deteriorated units.

Notations

$I(t)$ the inventory level at time t.

θt variable rate of defective units out of on hand inventory at time t, $0 < \theta << 1$.

C' the inventory ordering cost per order.

C_1 are the holding cost per unit per unit time

C_2 unit purchase cost per unit

C_3 shortage cost per unit per unit time

C_4 lost sale cost per unit per unit time

t_1 is the time at which shortage starts and T is the length of replenishment cycle. $0 \le t_1 \le T$.

$$f(t) = a + bt$$

The variable demand rate is, a > 0, b > 0.
Here a is initial rate of demand, b is the rate with which the demand rate increases.
$\exp(-\delta t)$ Unsatisfied demand is backlogged at a rate, the backlogging parameter δ is a positive constant.

FORMULATION AND SOLUTION OF THE MODEL

The depletion of inventory during the interval (0, t_1) is due to joint effect of demand and deterioration of items and the demand is partially backlogged in the interval (t_1, T). The differential equations describing the inventory level I (t) in the interval (0, T) are given by

$$I'(t) + \theta t\ I(t) = -f(t),\ 0 \le t \le t_1 \quad \ldots (1)$$

$$I'(t) = -f(t)\ e^{-\delta t}, t_1 \le t \le T \quad \ldots (2)$$

with the conditions, I (t_1) = 0 and I (0) = S ... (3)

The solutions of equations (1) and (2) can be obtained as

$$I(t) = a(t_1 - t) + \frac{b}{2}(t_1^2 - t^2) + \frac{a\theta}{6}(t_1^3 - 3t_1t^2 + 2t^3) + \frac{b\theta}{8}(t_1^2 - t^2)^2, 0 \le t \le t_1 \quad \ldots (4)$$

and $$I(t) = \left\{a\delta^2 + b\delta(\delta t + 1)\right\}\frac{e^{-\delta t}}{\delta^3} - \left\{a\delta^2 + b\delta(\delta t_1 + 1)\right\}\frac{e^{-\delta t_1}}{\delta^3},\ t_1 \le t \le T \quad \ldots (5)$$

Also the initial inventory level

$$S = at_1 + \frac{b}{2}t_1^2 + \frac{a\theta}{2}\frac{t_1^3}{3} + \frac{b\theta t_1^4}{8} \quad \ldots (6)$$

The inventory holding cost (C_H) per cycle is given by

$$C_H = C_1\int_0^{t_1} I(t)dt = C_1\left(\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{a\theta t_1^4}{12} + \frac{b\theta t_1^5}{15}\right) \quad \ldots (7)$$

The deterioration cost (C_D) per cycle is given by

$$C_D = C_2\left\{I(0) - \int_0^{t_1} f(t)dt\right\} = C_2\left\{\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8}\right\} \quad \ldots (8)$$

The shortage cost (C_S) per cycle due to backlog is given by

$$C_S = -C_3\int_{t_1}^{T} I(t)dt = \frac{C_3}{\delta^4}\left\{a\delta^2 + b\delta(2 + \delta T)\right\}e^{-\delta T}$$

$$-\frac{C_3}{\delta^4}\left[a\delta^2\left\{1 - \delta(T - t_1)\right\} + b\delta\left\{(2 - \delta T)(1 + \delta t_1) + \delta^2 t_1^2\right\}\right]e^{-\delta t_1} \quad \ldots (9)$$

and the opportunity cost (C_0) per cycle due to lost sales is given by

$$C_0 = C_4\int_{t_1}^{T}(1 - e^{-\delta t})(a + bt)dt$$

$$= C_4\left[a(T - t_1) + \frac{b}{2}(T^2 - t_1^2) + \frac{1}{\delta^3}\left\{a\delta^2 + b\delta(1 + \delta T)\right\}e^{-\delta T}\right.$$

$$\left. -\frac{1}{\delta^3}\left\{a\delta^2 + b\delta(1 + \delta t_1)\right\}\right]e^{-\delta t_1} \quad \ldots (10)$$

Hence, the total average cost of the system is given by

$$TC = \frac{1}{T}(C' + C_H + C_D + C_S + C_O) \quad \ldots (11)$$

$$= \frac{1}{T}[C' + C_1\left(\frac{at_1^2}{2} + \frac{bt_1^3}{3} + \frac{a\theta t_1^4}{12} + \frac{b\theta t_1^5}{15}\right) + C_2\left\{\frac{a\theta t_1^3}{6} + \frac{b\theta t_1^4}{8}\right\}$$

$$+\frac{C_3}{\delta^4}\left\{a\delta^2 + b\delta(2+\delta T)\right\}e^{-\delta T}$$

$$-\frac{C_3}{\delta^4}\left[a\delta^2\left\{1-\delta(T-t_1)\right\} + b\delta\left\{(2-\delta T)(1+\delta t_1) + \delta^2 t_1^2\right\}\right]e^{-\delta t_1}$$

$$+C_4[a(T-t_1) + \frac{b}{2}(T^2 - t_1^2) + \frac{1}{\delta^3}\left\{a\delta^2 + b\delta(1+\delta T)\right\}e^{-\delta T}$$

$$-\frac{1}{\delta^3}\left\{a\delta^2 + b\delta(1+\delta t_1) + c(2 + 2\delta t_1 + \delta^2 t_1^2)\right\}e^{-\delta t_1}]]$$

To minimize total average cost per unit time, the optimal values of t_1 and T can be obtained by solving the following equations simultaneously

$$\frac{\partial TC}{\partial t_1} = 0 \quad \ldots (12)$$

and $$\frac{\partial TC}{\partial T} = 0 \quad \ldots (13)$$

provided they satisfy the following conditions

$$\left.\begin{array}{l} \dfrac{\partial^2 TC}{\partial t_1^2} > 0, \dfrac{\partial^2 TC}{\partial T^2} > 0 \\ \text{and} \quad \left(\dfrac{\partial^2 TC}{\partial t_1^2}\right)\left(\dfrac{\partial^2 TC}{\partial T^2}\right) - \left(\dfrac{\partial^2 TC}{\partial t_1 \partial T}\right)^2 > 0 \end{array}\right\} \quad \ldots (14)$$

The numerical solution of these equations can be obtained by using some suitable computational numerical method.

CONCLUSION

In this paper, we allow the shortages with partial backlogging in this model and backlogging rate is an exponential decreasing function of time. From the analysis of model, it has been concluded that if the demand parameters are increases then the time and total cost are increases. If the deterioration parameter is increases then the time is decreases and total cost is increases. A natural extension of this research is to consider finite replenishment.

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